R_T and r_T, steady-state and the transient thermal resistance of the transitor in the plane of power generation; R_{TP} and r_{TP}, steady-state and the transient thermal resistance of the PTC thermistor; y_P, transfer function of the PTC thermistor with respect to temperature; U_C and u_C, dc and the instantaneous collector voltage in the transistor circuit; U_{in} and u_{in}, dc and the instantaneous input voltage to the transistor stage; I_{C0}, dc collector current in the transistor corresponding to $\vartheta(x, t) \leq \Theta_{cr}$; i_C, instantaneous collector current; i_b, instantaneous base current; R_P and r_P, steady-state and the instantaneous electrical resistance of the PTC thermistor; γ, R_0, R_d, parameters of the resistance-temperature characteristic of the PTC thermistor; ρ_P, electrical resistivity of the PTC thermistor; σ_P, cross-sectional area of the PTC thermistor; l_P, thickness of the PTC thermistor in the x direction; and β, a, k, c, b_1, b_2, coefficients.

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CALCULATION OF A TRANSIENT IN A TWO-POLE NETWORK WITH A THERMISTOR

I. M. Trushin and V. I. Antonov

UDC 536.63:621.316.825:621.3.011.1

An analytical expression is derived which describes the variation of the thermistor temperature during a relay-effect transient process following an instantaneous change in the supply voltage in an $R - R_T$ two-pole network.

Thermistors, a widely known class of semiconductor devices, can be successfully used in various relay and pulse devices utilizing the electrothermal relay effect. This effect takes place when a dc voltage is applied to the input of two-pole network containing a thermistor and a linear resistor [1-5].

The transient process in such a network will be described by the well-known differential equation [1]

$$C_V \frac{dT}{dt} = \frac{E^2 R_\infty \exp(B/T)}{[R_\infty \exp(B/T) + R]^2} - H(T - T_a). \tag{1}$$

This equation can be easily reduced to quadratures by separation of variables, but the resulting expression in terms of elementary function is not integrable. For this reason, several methods of simplifying the fundamental equation (1) have been developed so as to yield a solution. These include linearization of the differential equation (1), assuming small deviations of thermal and electrical parameters in the network, the method of piecewise-linear approximation [3], replacement of the thermistor with an equivalent two-pole network [4], graphical integration [5], etc. However, these methods either are applicable to only a narrow temperature range or require special graph plotting without being universal and convenient.

In this study, based on a set of assumptions about the characteristics of electrothermal processes in a thermistor, an attempt will be made to simplify Eq. (1) and to solve it in an analytical form.

For the construction of a workable physical model describing the processes of charge transfer in a thermistor, we will utilize the fact that the current–voltage characteristic of a thermistor has a typical, for it, range of negative resistance. The physical processes occurring in semiconductor devices with such a characteristic are conveniently described with the aid of models which utilize concepts pertaining to a so-called hot gas of charge carriers [6].

Let us examine the process of current flow in a thermistor on the basis of these concepts. After a voltage has been applied to the input of a four-pole network containing a thermistor and a linear resistor, the

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electric field imparts to charge carriers in the thermistor additional kinetic energy. The kinetic energy acquired by charges in a unit volume during a unit of time is [6]

$$P_{TK} = n_e \frac{2/3 \cdot E_d}{\tau_e} + n_e \frac{k(T_e - T_L)}{\tau_e} = P_{LK} + P_{ek}. \tag{2}$$

The potential energy which the electric field imparts to the thermistor at the first instant of time is converted partly to Joule-effect heat and partly to "heating" the gas of charge carriers, viz.,

$$P_0 \equiv nP_0 + (1-n)P_0 = P_{e0} + P_{L0}. \tag{3}$$

Therefore, the total energy acquired by the thermistor per unit time is

$$P_T = P_{TK} + P_0 = P_{e0} + P_{eK} + P_{L0} + P_{LK} \tag{4}$$

Substituting this expression for the first term on the right-hand side of Eq. (1) yields

$$C_V \frac{d\Delta T}{dt} = P_{e0} + P_{eK} + P_{L0} + P_{LK} - H\Delta T. \tag{5}$$

We will now express P_{eK} and P_{LK} through the temperature drops (ΔT_e) in the "gas of charge carriers—crystal lattice" system and (ΔT_L) in the "crystal lattice—ambient medium" system, respectively.

For the "gas of charge carriers—crystal lattice" system relation (2) yields directly

$$P_{eK}(\Delta T_e) = N_1 \Delta T_e. \tag{6}$$

For establishing the $P_{LK}(\Delta T_L)$ relation we consider that the temperature of the crystal lattice is related to the thermistor conductance according to the well-known expression

$$Y_T = \frac{1}{R_\infty} \exp(-B/T). \tag{7}$$

Expanding this function into a Taylor series in the vicinity of the initial temperature and retaining only the first two terms of this expansion, we have

$$\Delta Y_T = Y_T - Y_0 \cong \beta_T Y_0 \Delta T_L. \tag{8}$$

Considering that the current in the circuit containing a thermistor and a linear resistor is

$$I_L = \frac{EY_T}{1 + Y_T R}, \tag{9}$$

and assuming that $Y_T R \ll 1$, we rewrite expression (9) on the basis of relation (8) as

$$\Delta I_L = I_L - I_0 \cong N_2 \Delta T_L. \tag{10}$$

Therefore

$$P_{LK}(\Delta T_L) = EN_2 \Delta T_L - RN_2^2 \Delta T_L^2. \tag{11}$$

Noting that $\Delta T = \Delta T_e + \Delta T_L$ and using the relations (3), (6), (11), we rewrite Eq. (5) as a system of three equations

$$\begin{aligned} C_V \frac{d\Delta T_e}{dt} &= nP_0 + (N_1 - H)\Delta T_e; \\ C_V \frac{d\Delta T_L}{dt} &= (1-n)P_0 - RN_2^2 \Delta T_L^2 + (EN_2 - H)\Delta T_L; \\ \Delta T &= \Delta T_e + \Delta T_L \end{aligned} \tag{12}$$

The solution to these differential equations for the given initial conditions ($\Delta T_{e0} = \Delta T_{L0} = 0$ at $t = 0$) is

$$\Delta T_L = \frac{\sqrt{(N_2E - H)^2 + 4R(1-n)P_0N_2^2}}{2RN_2^2} \operatorname{th}\left[\frac{\sqrt{(N_2E-H)^2 + 4R(1-n)P_0N_2^2}}{2C_V} t - \operatorname{Arth} \frac{N_2E - H}{\sqrt{(N_2E-H)^2 + 4R(1-n)P_0N_2^2}}\right] + \frac{N_2E - H}{2RN_2^2}; \tag{13}$$

$$\Delta T_e = \frac{nP_0}{H - N_1}\left(1 - \exp\left(\frac{N_1 - H}{C_V}\right)t\right). \tag{14}$$

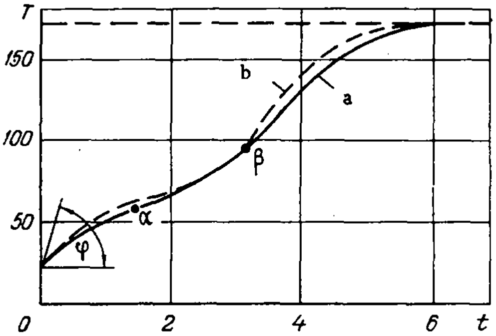

Fig. 1. Transient process in the $R-R_T$ network (grade MT-57 thermistor) after application of the supply voltage: (a) theoretical curve based on relations (13) and (14); (b) experimental curve; T (°C), t (sec).

The curve in Fig. 1 depicts the transient process $\Delta T(t) = \Delta T_e + \Delta T_L$, plotted according to relations (13) and (14). The trend of this transient curve can, evidently, be characterized by the slope angle at the beginning, inflections at points α and β, and a horizontal asymptote at $t \to \infty$.

In addition to the known network parameters (P_0, H, C_V, R, E), there also appear in Eqs. (13) and (14) the quantities n, N_1, and N_2 with not yet determined values, We will select the values of parameters n, N_1, and N_2 so as to make the right-hand side of Eq. (1) equal to the sum of the right-hand sides of the differential equations in system (12) at certain characteristic points. For this purpose, we must first analyze the graph of the function which corresponds to Eq. (1) (Fig. 2).

On this graph we note four characteristic points: A, B, C, D. The coordinates of point A are $A(0, P_0)$. Since $\Delta T \geq 0$ and $\Delta T_L \geq 0$, hence obviously the condition $\Delta T_0 = \Delta T_{e0} + \Delta T_{L0}$ yields $\Delta T_{e0} = 0$ and $\Delta T_{L0} = 0$. Inserting these values into the system of equations (12), we obtain

$$C_V \frac{d\Delta T_{L0}}{dt} + C_V \frac{d\Delta T_{e0}}{dt} = nP_0 + (1-n)P_0 = P_0. \tag{15}$$

In this way, the selection of the initial conditions here ensures that at point A the right-hand side of Eq. (1) is equal to the sum of the right-hand sides of Eqs. (12).

Point D in Fig. 2 corresponds to thermal steady state in the thermistor at $t \to \infty$. Under these conditions $d\Delta T/dt = 0$. At $t \to \infty$, furthermore, $d\Delta T_e/dt = 0$ and $d\Delta T_L/dt = 0$, inasmuch as Eqs. (13) and (14) have here horizontal asymptotes. Consequently, at point D there must be satisfied the conditions

$$\begin{aligned} -nP_0 &= (N_1 - H)\Delta T_{e\,\max}; \\ -(1-n)P_0 &= (EN_2 - H)\Delta T_{L\,\max} - RN_2^2 \Delta T_{L\,\max}^2. \end{aligned} \tag{16}$$

Considering that $\Delta T_{max} = \Delta T_{e\,max} + \Delta T_{L\,max}$, we find n from system (16):

$$n = \frac{H - N_1}{P_0}\left(\Delta T_{\max} - \frac{(N_2E - N_1)}{2RN_2^2} + \sqrt{\left(\frac{N_2E - N_1}{2RN_2^2}\right)^2 + \frac{P_0 + (N_1 - H)\Delta T_{\max}}{RN_2^2}}\right). \tag{17}$$

In order to ensure that the right-hand side of Eq. (1) is equal to the sum of the right-hand sides of Eqs. (12) at points B and C, it is necessary to differentiate Eq. (1) with respect to time and let $d^2\Delta T/dt^2 = 0$, inasmuch as these points correspond to the inflection points α and β in Fig. 1. As a result, since $d\Delta T/dt > 0$ over the entire temperature range $(0, \Delta T_{max})$,

$$y = \frac{x(x-\gamma)\ln^2 x}{(x+\gamma)^3}, \tag{18}$$

with x, y, and γ denoting the respective ratios

$$x = \frac{R_{T,\,cr}}{R_\infty}, \quad y = \frac{HBR_\infty}{E^2}; \quad \gamma = \frac{R}{R_\infty}.$$

The family of functions which correspond to expression (18) with various values of γ is shown in Fig. 3. Here points L(L', L") and Q correspond, respectively, to the beginning and the end of the transient process (straight-line segment Q–L).

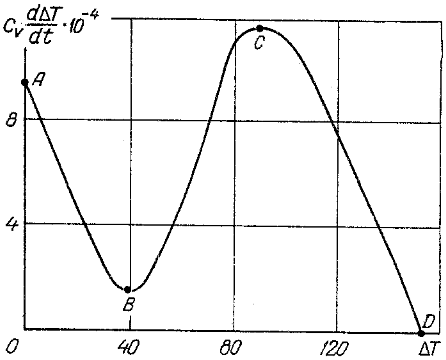

Fig. 2

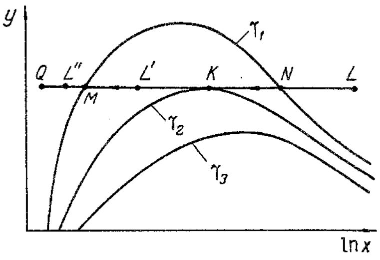

Fig. 3

Fig. 2. Power $C_V(d/\Delta T/dt) \cdot 10^{-4}$ (W) expended on changing the heat content in a thermistor (grade MT-57) as a function of the instantaneous temperature drop ΔT (°C), according to Eq. (1).

Fig. 3. Graph for the solution of Eq. (18); x, y, γ are dimensionless quantities.

An analysis of these graphs indicates three possible solutions to Eq. (18):

1) Eq. (18) has two roots (points M and N on curve $\gamma = \gamma_1$), which means that the transient curve (Fig. 1) has two inflection points;

2) Eq. (18) has one root (point K on curve $\gamma = \gamma_2$), which means that the two inflection points on the transient curve merge into one;

3) Eq. (18) has no roots (curve $\gamma = \gamma_3$), which means that the transient process evolves without inflection points.

We are interested in the first case only, because here the transient curve has two inflection points and, therefore, the relay effect can occur.

Noteworthy is the possibility that, depending on the selection of the point where the transient process begins (L' or L'', for example), the temperature T_0 at the beginning of the transient process can exceed the temperature at one inflection point (point N) or both temperatures at the two inflection points (M and N). This means that the transient process will evolve with either one or no inflection point. Even in this case, however, it can be tentatively regarded as a relay-effect process, but as one which begins above point α or β. Consequently, the subsequent analysis applies to this case too.

It is possible to determine from Eq. (18) both ΔT_{crB} and ΔT_{crC} and from Eq. (1) both $d\Delta T_{crB}/dt$ and $d\Delta T_{crC}/dt$ at these respective points. According to the stated object of these calculations, it is necessary to ensure that the right-hand side of Eq. (1) will be equal to the sum of the right-hand side of Eqs. (12) at points B and C. This requirement will be satisfied when at these points

$$C_V \frac{d^2\Delta T_{cr}}{dt^2} = C_V \frac{d^2\Delta T_{e,cr}}{dt^2} + C_V \frac{d^2\Delta T_{L,cr}}{dt^2} = 0. \quad (19)$$

Differentiating the equations in system (12) with respect to time yields

$$\begin{aligned} C_V \frac{d^2\Delta T_{e,cr}}{dt^2} &= C_V (N_1 - H) \frac{d\Delta T_{e,cr}}{dt}, \\ C_V \frac{d^2\Delta T_{L,cr}}{dt^2} &= C_V (EN_2 - H - 2RN_2^2) \frac{d\Delta T_{L,cr}}{dt}, \\ \frac{d\Delta T_{cr}}{dt} &= \frac{d\Delta T_{e,cr}}{dt} + \frac{d\Delta T_{L,cr}}{dt}. \end{aligned} \quad (20)$$

With the aid of condition (19) we finally obtain

$$(N_1 - H)[(N_1 - H)\Delta T_{e,cr} + nP_0] = [2RN_2^2(\Delta T_{cr} - \Delta T_{e,cr}) - EN_2 + H]\left[C_V \frac{d\Delta T_{cr}}{dt} - (N_1 - H)\Delta T_{e,cr} - nP_0\right]. \quad (21)$$

Here $\Delta T_{e,cr}$ is a root of the quadratic equation

$$C_V \frac{d\Delta T_{cr}}{dt} = (N_1 - H)\,\Delta T_{e,\,cr} + (EN_2 - H)(\Delta T_{cr} - \Delta T_{e,\,cr}) - (\Delta T_{cr} - \Delta T_{e,\,cr})^2 RN_2^2 + P_0. \tag{22}$$

Since Eq. (21) must be satisfied at the two points $B(\Delta T_{crB}, d\Delta T_{crB}/dt)$ and $C(\Delta T_{crC}, d\Delta T_{crC}/dt)$, hence upon insertion of their coordinates into this equation the latter will be split into two equations, which together with expression (17) form a system of three equations with three unknowns n, N_1, and N_2:

$$\begin{gathered} n = \frac{H - N_1}{P_0}\left(\Delta T_{max} - \frac{EN_2 - N_1}{2RN_2^2} + \sqrt{\left(\frac{EN_2 - N_1}{2RN_2^2}\right)^2 + \frac{P_0 + (N_1 - H)\,\Delta T_{max}}{RN_2^2}}\right), \\ (N_1 - H)\,[(N_1 - H)\,\Delta T_{e,crB} + nP_0] = [2RN_2^2(\Delta T_{crB} - \Delta T_{e,crB}) - EN_2 + H]\left[C_V \frac{d\Delta T_{crB}}{dt} - (N_1 - H)\,\Delta T_{e,crB} - nP_0\right], \\ (N_1 - H)\,[(N_1 - H)\,\Delta T_{e,crC} + nP_0] = [2RN_2^2(\Delta T_{crC} - \Delta T_{e,crC}) - EN_2 + H]\left[C_V \frac{d\Delta T_{crC}}{dt} - (N_1 - H)\,\Delta T_{e,crC} - nP_0\right], \end{gathered} \tag{23}$$

where $\Delta T_{e,crB}$ and $\Delta T_{e,crC}$ are found from Eq. (22).

Having determined n, N_1, and N_2 from this system, we obviously obtain an analytical function T(t) for expressions (13) and (14) which approximately describes a relay-effect transient process in a network with a thermistor and also ensures that Eq. (1) will be identical to the system of Eqs. (12) at point A where the transient process begins, at the inflection points B and C, and at point D where the transient process ends. For estimating the accuracy of this approximation we can use the expression [7]

$$|\Delta T(t) - \Delta T_e(t) - \Delta T_L(t)| \leqslant \frac{\delta}{M}\,(\exp(M \cdot |t - t_0|) - 1). \tag{24}$$

We must note that the proposed physical model of electrothermal processes occurring in a thermistor is a tentative one and has been used by these authors only within the framework of the specific formulated problem. Estimating how close this model corresponds to the real processes of transfer of electric charges in a thermistor is difficult, because the nature of electrical conductivity of 3-d oxides (including thermistors) has to this time not yet been thoroughly enough explored [8].

The proposed method of calculating a transient in a network containing a thermistor and a linear resistor after application of voltage E has been checked on a network containing a grade MT-57 thermistor (R_∞ = 0.1865 Ω, B = 3148°K, H = $7 \cdot 10^{-5}$ W/deg C, C_V = $1.6 \cdot 10^{-5}$ W · sec/deg C, T_a = 21°C) and a linear resistor (R = 190.9 Ω) under a supply voltage E = 2.86 V.

A curve depicting the transient process was plotted on the basis of calculations (curve a in Fig. 1). For comparison of these theoretical results with experimental data, on the same diagram is shown curve b depicting the transient process according to oscillograms. There appears to be a close qualitative and quantitative agreement between curves a and b. This confirms the appropriateness of using the proposed method for such important practical problems as design of thermistor delay lines and time relays. The method can also be found useful for analysis of transient processes in networks containing a linear capacitance or inductance, also in other networks.

NOTATION

E, supply voltage applied to the two-pole network, V; R, linear resistance in this network, Ω; R_T, thermistor resistance at temperature T, Ω; R_∞, static resistance of the thermistor at t → ∞, Ω; $R_{T,cr}$, thermistor resistance at an inflection point, Ω; B, activation temperature for charge carriers, °K; T_a, ambient temperature, °K; P_T, power generated in the thermistor, W; P_{TK}, kinetic energy acquired by charges in a unit volume during a unit time, W; n_e, concentration of charge carriers in the thermistor; E_d, drift energy of charge carriers, J; k = $1.38 \cdot 10^{-23}$ J/°K, Boltzmann's constant; τ_e, energy relaxation time, sec; P_{eK}, kinetic energy acquired by the gas of charge carriers per unit time, W; P_{LK}, kinetic energy acquired by the crystal lattice per unit time, W; P_0, power dissipated in the thermistor at the first instant of time, W; P_{e0}, potential energy expended on heating the gas of charge carriers at the first instant of time, W; P_{L0}, potential energy expended on generating Joule-effect heat at the first instant of time, W; n = P_{e0}/P_0, a proportionality factor; ΔT, temperature drop from thermistor to ambient medium, °C; ΔT_e, temperature drop from gas of charge carriers to crystal lattice, °C; ΔT_L, temperature drop from crystal lattice to ambient medium, °C; Y_T, thermistor conductance at temperature T, mho; Y_0, initial thermistor conductance, mho; β_T, temperature coefficient of thermistor resistance at temperature T_0, 1/K; I_L, current in the network which generates Joule-effect heat, A; I_0, initial current in the network which generates Joule-effect heat at the first instant of time, A; N_1, a proportionality factor, W/°C; N_2, a proportionality factor, A/°C; $\Delta T_{e,max}$, maximum temperature drop from gas of charge carriers to crystal lattice, °C; $\Delta T_{L,max}$, maximum temperature drop from crystal lattice to ambient medium, °C; ΔT_{max}, maximum temperature drop from thermistor to ambient medium, °C; ΔT_{crB} and ΔT_{crC}, temperature drops corresponding to points B and C, respectively, °C; $d\Delta T_{crB}/dt$ and $d\Delta T_{crC}/dt$, numerical values of the derivative at points B and C, respectively, °C/sec; δ, a constant [7]; M, Lipschitz constant; t, time, sec; H, dissipation coefficient of the thermistor, W/°C; and C_V, volumetric heat capacity of the thermistor, W·sec/°C.

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EFFECT OF THE GEOMETRY OF THE MOVING WALL ON THE STRUCTURE OF THE FLOW IN CONFINED FLOW IN A SLIT GAP

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UDC 532.516

An analytical solution of the problem is presented with an estimate of the effect of the wavy oscillating wall on the flow characteristics of a viscous liquid in a slit gap.

When analyzing the heat- and mass-exchange characteristics between a flow of liquid and a solid surface one must bear in mind that the geometry of the channel walls has a considerable influence on the structure of the flow. Experimental methods are widely used to solve this problem because of the mathematical difficulties. In [1] an estimate is made of the effect of the microgeometry of the surface on the structure of the flow based on a solution of the problem of Couette flow with a fixed wavy wall.

We will consider the more general nonstationary case when the wavy wall performs harmonic oscillations, the flow is confined, and the gap between the walls h is fairly small compared with the characteristic length of the channel. The Reynolds number is assumed to be very much less than 1.

The law of motion of the lower wall and the equations of the upper and lower walls can be written in the form

$$x_t = x_0 + a\sin\omega t;$$
$$y_l(x,\ t) = \bar{e}\sin k(x - a\sin\omega t); \qquad (1)$$
$$y_u = h = \text{const},$$

where a and $\bar{e}$ are the amplitudes of oscillation of the wall and the wavy surface; k, wave number; and ω,

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